Mechanical Stiffness and Malleability of Hard Cheese

A Rheological Study on the Viscoelastic Properties of Aged Cheese Varieties

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Abstract

The mechanical behavior of hard cheese varieties is a key determinant of their processing characteristics, textural perception, and overall quality. This study investigates the stiffness and malleability of aged cheeses by combining rheological theory and experimental measurements. A viscoelastic constitutive model is used to describe the dependence of the elastic modulus on strain rate and temperature. Compression tests were conducted on Parmigiano-Reggiano, Comté, and aged Cheddar under controlled conditions. The results demonstrate significant variability among cheese types, primarily influenced by moisture and fat content. Computational modeling using Python highlights the predictive capability of the proposed formulation. The findings provide a quantitative framework for linking the microstructure of hard cheeses to their macroscopic mechanical response.

Introduction

Cheese, though traditionally perceived as a food product, can also be regarded as a **soft viscoelastic solid**, whose rheological response depends on both time and temperature. The interplay between its **elastic stiffness** — resistance to deformation — and **malleability** — the capacity to deform without fracture — defines its sensory and functional attributes [2].

The mechanical performance of hard cheeses arises from their intricate **protein–fat–moisture matrix**. Protein cross-linking, crystalline calcium phosphate domains, and residual fat globules act as reinforcing or plasticizing agents depending on aging and composition [1]. Understanding these parameters is crucial not only for consumer perception (texture, crumble, brittleness) but also for industrial operations such as slicing, shredding, and packaging.

This work presents an integrated experimental–computational approach to quantify the rheological stiffness and malleability of hard cheese varieties, using an empirical viscoelastic model to rationalize observed behavior across temperature and strain-rate ranges.

Theoretical Framework

The elastic modulus E of a viscoelastic food material is modeled as a function of strain rate $\dot{\varepsilon}$ and temperature T:

$$E(T, \dot{\varepsilon}) = E_0 \left(1 - \alpha(T - T_0)\right) \left(1 + \beta \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)$$

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where:

- E_0: reference modulus at temperature T_0,
- α: thermal softening coefficient,
- β: strain-rate sensitivity coefficient.

Equation (1) assumes linear viscoelasticity and moderate deformation, conditions generally valid below the yield point of hard cheese. Increasing temperature reduces stiffness via thermally activated molecular mobility, whereas higher strain rates enhance stiffness due to time-dependent stress relaxation suppression.

The **malleability**, defined here as the reciprocal of stiffness, becomes:

$$M = \frac{1}{E(T, \dot{\varepsilon})}$$

This formalism provides a convenient means to estimate the flexibility of cheese under various thermal–mechanical conditions, offering a bridge between empirical texture measurements and predictive modeling.

Melting and Thermal Behavior

The melting characteristics of cheese are governed primarily by **fat content**, **moisture level**, and the degree of **protein–fat network entanglement**. Hard cheeses typically display a melting range between **50 °C and 70 °C**, depending on composition and aging.

Thermal analysis using **Differential Scanning Calorimetry (DSC)** allows the determination of onset and peak melting temperatures. In contrast, high-moisture cheeses such as Mozzarella [1]stretchability due to reduced protein cross-link density [3]. The figure illustrates the melting behavior of Mozzarella cheese, highlighting its unique rheological response upon heating.

[1]A high-moisture cheese known for its stretchability when melted.

Figure 1: Melting behavior of high-moisture cheese (Mozzarella) illustrating stretchability upon heating

Materials and Methods

Samples Preparation

Three commercial hard cheese types were selected for analysis . Cylindrical specimens (20 mm diameter × 20 mm height) were extracted from the core regions and equilibrated at **10 °C** for 12 h prior to testing to minimize moisture gradients.

Table 1: Characteristics of hard cheese samples used in the study

Cheese Type	**Age (months)**	**Moisture Content (%)**	**Fat Content (%)**
Parmigiano-Reggiano	24	29	31
Comté	18	33	30
Cheddar	12	36	32

Mechanical Testing

Compression tests were carried out using a **TA-XT2 texture analyzer** equipped with a 50 kg load cell. Specimens were compressed at three nominal strain rates (0.1, 1, and 10 s^{-1}) to 20 % strain. The **elastic modulus (E)** was obtained from the initial linear region (0–5 % strain) of the stress–strain curve. The table summarizes the measured moduli.

Table 3: Measured elastic moduli of hard cheese samples at varying strain rates

Cheese Type	**Strain Rate (s^{-1})**	**Temperature (°C)**	**Elastic Modulus E (MPa)**
Parmigiano-Reggiano	0.1	10	52.1
Parmigiano-Reggiano	10	10	66.8
Comté	0.1	10	44.2
Cheddar	0.1	10	38.7

The increasing modulus with strain rate reflects the time-dependent viscoelastic response typical of aged dairy matrices.

Computational Modeling

To illustrate the relationship between stiffness, temperature, and deformation rate, the theoretical expression (Eq. 1) was implemented in **Python** as follows:

```python
import matplotlib.pyplot as plt
import numpy as np
import numpy.typing as npt

# Constants
E0 = 50.0  # Reference modulus (MPa)
T0 = 10.0  # Reference temperature (°C)
alpha = 0.02  # Thermal softening coefficient
beta = 0.15  # Strain-rate sensitivity
eps_dot0 = 1.0  # Reference strain rate

ArrayLike = npt.NDArray[np.float64] | float

```

```python
def modulus(temperature: ArrayLike, strain_rate: float) -> ArrayLike:
    """Return elastic modulus as a function of temperature and strain rate."""
    return E0 * (1 - alpha * (temperature - T0)) * (1 + beta *
        np.log(strain_rate / eps_dot0))

# Compute modulus for a range of temperatures
temperatures = np.linspace(5, 25, 100)
rates = [0.1, 1, 10]

plt.figure(figsize=(6, 4))
for r in rates:
    plt.plot(temperatures, modulus(temperatures, r), label=f"$\dot{\epsilon}$ = {r} s$^{-1}$")
plt.xlabel("Temperature (°C)")
plt.ylabel("Elastic Modulus E (MPa)")
plt.title("Temperature Dependence of Cheese Stiffness")
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()
```

This computational approach allows the parametric exploration of cheese stiffness under varying conditions, offering predictive insight into texture control during processing.

Discussion

The results corroborate the expected hierarchy of stiffness among hard cheeses, with **Parmigiano-Reggiano** exhibiting the highest elastic modulus, consistent with its lower moisture and greater protein cross-linking. The **Comté** sample demonstrated intermediate stiffness, while **Cheddar**, being relatively younger and moister, showed greater malleability.

The positive strain-rate dependence (via $\beta > 0$) implies that cheese behaves more elastically under rapid deformation, an important consideration for high-speed industrial slicing. Conversely, the temperature dependence ($\alpha > 0$) highlights the need for strict temperature control during mechanical handling to maintain structural integrity.

The modeling results align qualitatively with the empirical data, suggesting that the simplified rheological model captures the dominant trends despite the inherent complexity of cheese microstructure.

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Conclusions

This study integrates rheological experimentation and computational modeling to quantify the **stiffness–malleability balance** of hard cheeses. The proposed formulation effectively predicts the combined influence of **temperature** and **strain rate** on the elastic modulus.

Such predictive tools can aid in optimizing industrial cheese handling, from cutting and packaging to consumer preparation, by linking measurable mechanical parameters to sensory texture and thermal stability.

Future work should extend the model to include **nonlinear viscoelasticity** and **moisture-dependent plasticization effects**, as well as **microstructural imaging (e.g., SEM, CLSM)** to directly correlate morphology with rheological behavior.

References

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